

Remember that :-

$$\int_a^b f(x) dx = \lim_{n \rightarrow \infty} \sum_{k=1}^n \frac{b-a}{n} f\left(a + \frac{b-a}{n} k\right).$$

Theorem :-

Pg 80

If $f(x)$ is a continuous fn on $[a, b]$, then there exists a point $c \in [a, b]$

s.t. $\left\{ \frac{1}{b-a} \int_a^b f(x) dx = f(c) \right\}$

Some properties of Definite Integral :-

If $f(x)$, $g(x)$ are continuous fns on $[a, b]$

$a \in [a, b]$, const

$$1 - \int_a^b [F(x) \pm g(x)] dx = \int_a^b F(x) dx \pm \int_a^b g(x) dx$$

$$2 - \int_a^b c F(x) dx = c \int_a^b F(x) dx$$

$$3 - \int_a^b F(x) dx = \int_a^d F(x) dx + \int_d^b F(x) dx$$

$$4 - \int_a^b F(x) dx = - \int_b^a F(x) dx$$

$$5 - \int_a^a F(x) dx = \text{Zero}$$

$$6 - \text{If } F(x) \leq g(x) \Rightarrow \int_a^b F(x) \leq \int_a^b g(x)$$

$$\forall \text{ all } x \in [a, b]$$

$F(x)$ odd function

$$7 - \int_{-a}^{+a} F(x) dx = \text{Zero:}$$

$$8 - \int_{-a}^{+a} F(x) dx = 2 \int_0^a F(x) dx$$

$F(x)$ even

Theorem (2) :-

Let $f(x)$ be a continuous fn on $[a, b]$

IF $x \in [a, b]$

and $\int_a^x f(x) dx = F(x)$

$f(x)$ must be a continuous on $[a, b]$

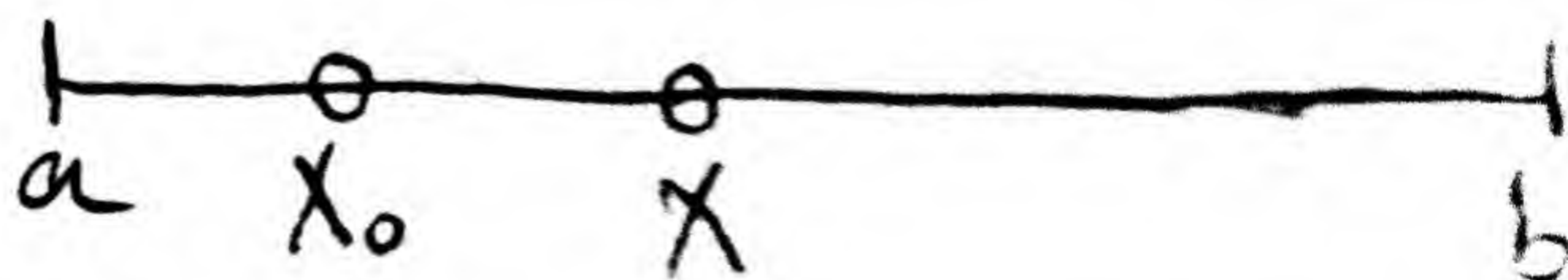
and differentiable on (a, b)

, then ??

$$\boxed{F'(x) = f(x)}$$

prove of theorem 2

$$\frac{f(x) - f(x_0)}{x - x_0}$$



$$= \frac{1}{x - x_0} \left\{ \int_a^x f(x) dx - \int_a^{x_0} f(x) dx \right\}$$

$$= \frac{1}{x - x_0} \int_{x_0}^x f(x) dx$$

$$= f(c), \quad c \in [x_0, x]$$

taking the limit

$$\text{as } x \rightarrow x_0$$

then we get:-

$$f'(x_0) = \lim_{x \rightarrow x_0} f(x) = f(x_0)$$

Since x_0 is arbitrary point $\in [a, b]$. Then:-

$$f'(x) = f(x) \quad \forall \text{ all } x \in [a, b]$$

Theorem:

the Fundamental theorem of calculus:-

$$\int_a^b f(x) = f(b) - f(a)$$

$$\int_1^4 x \, dx = \left. \frac{x^2}{2} \right|_1^4 = \frac{4^2}{2} - \frac{1^2}{2}$$

$$\int f(x) \, dx = f(x) + C$$

Ex 2 Pg 2

$$(1) ? \frac{d}{dx} [x^2 e^x]$$

Soln

$$[x^2 e^x]' = 2x e^x + x^2 e^x \rightarrow (1)$$

-1 Solo

$$(2) ? \int [2x e^x + x^2 e^x] dx = x^2 e^x + C \rightarrow (2)$$

$$(3) ? \int_1^5 [2x e^x + x^2 e^x] dx = x^2 e^x \Big|_1^5 = 5^2 e^5 - 1^2 e^1 = 25e^5 - e = 3507.6$$

$$\int x^n dx = \frac{x^{n+1}}{(n+1)} + C, n \neq -1$$

$$\int x^{-1} dx = \ln x + C$$

Table Pg 566 \leftarrow ~~here~~

$$\int a^x dx = \frac{a^x}{\ln a} + C$$

$$\int a^{bx} dx = \frac{a^{bx}}{b \ln a} + C$$

Pg 5:-

$$4 - \int \sin(ax) dx = -\frac{1}{a} \cos(ax) + C$$

$$6 - \int \cos(ax) dx = \frac{1}{a} \sin(ax) + C$$

$$8. \int \sec^2(ax) = \frac{1}{a} \tan(ax) + C$$

$$10. \int \operatorname{cosec}^2(ax) = -\frac{1}{a} \cot(ax) + C$$

$$17. \int \frac{1}{1+x^2} dx = \tan^{-1} x + C$$

$$18. \int \frac{1}{a^2+x^2} dx = \frac{1}{a} \tan^{-1}\left(\frac{x}{a}\right) + C$$

$$\text{ex. 5} \quad [a^u]' = u' a^u \ln a$$

Pg 8

$$I = \int \left[t^{\frac{2}{3}} + \frac{4}{5+t^2} - 3^{2t} \right] dt$$

$$I = \int_{I_1} t^{\frac{2}{3}} dt + \int_{I_2} \frac{4}{5+t^2} dt - \int_{I_3} 3^{2t} dt$$

$$I_1 = \int t^{\frac{2}{3}} dt = \frac{3t^{\frac{5}{3}}}{5}$$

$$I_2 = 4 \int \frac{1}{(\sqrt{5})^2 + t^2} dt = \frac{4}{\sqrt{5}} \tan^{-1} \frac{t}{\sqrt{5}}$$

$$I_3 = \int 3^{2t} = \frac{1}{2 \ln 3} \int 2 \times 3^{2t} \ln 3 dt = \frac{1}{2 \ln 3} \times 3^{2t}$$

$$I = \frac{3}{5} t^{\frac{5}{3}} + \frac{4}{\sqrt{5}} \tan^{-1} \frac{t}{\sqrt{5}} - \frac{1}{2 \ln 3} \times 3^{2t} + C$$

Find pg 9

$$a) \int \frac{x^2+1}{x} dx$$

$$b) \int \sin^4 x dx$$

Soln

$$a) I = \int \frac{x^2+1}{x} dx = \int \left(\frac{x^2}{x} + \frac{1}{x} \right) dx$$

$$= \int x dx + \int \frac{1}{x} dx = \frac{x^2}{2} + \ln|x| + C$$

$$b) \int \sin^4 x dx$$

Remember that

$$\cos 2x = 2\cos^2 x - 1$$

$$= 1 - 2\sin^2 x$$

$$\sin^2 x = \frac{1 - \cos 2x}{2}$$

$$\sin^4 x = \frac{1}{4} [1 - \cos 2x]^2 = \frac{1}{4} [1 - 2\cos 2x + \cos^2 2x]$$

$$\cos^2 x = \frac{\cos 2x + 1}{2}$$

$$\cos^2 2x = \frac{\cos 4x + 1}{2}$$

$$\sin^4 x = \frac{1}{4} \left[1 - 2\cos 2x + \frac{\cos 4x + 1}{2} \right]$$

Subject.

Date.

$$\int \sin^4 x \, dx = \int \frac{1}{4} \left[1 - 2\cos 2x + \frac{\cos 4x + 1}{2} \right] 2x$$

$$= \frac{1}{4} \left[x - 2 \frac{\sin 2x}{2} + \frac{1}{2} x + \frac{1}{2} \frac{\sin 4x}{4} \right] + C$$